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SCALE OF SUBJECTIVE NUMBER FOR INDIVIDUAL SUBJECTS

BY

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A THESIS

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The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies and Research, for acceptance, a thesis entitled "Scale of Subjective Number for Individual Subjects" submitted by Jerry M. Cinani in partial fulfilment of the requirements for the degree of Master of Science.

ABSTRACT

Two groups of 3 Ss compared the subjective size of circles with the integers 1 to 10. For one group the integers represented category labels. These Ss also made category judgments of the area of circles presented one at a time (single stimulus presentation). The Ss in the other group made comparisons of circle-number pairs when the integers represented numbers used in magnitude estimation and then made magnitude estimation judgments of the area of circles presented one at a time. Two scales were constructed for each S: one from the circle-number comparisons and one from the category rating or magnitude estimation responses. For the circle-number comparisons a Thurstone-type discriminability scale of number was obtained from the relative frequency with which each number was judged greater than the subjective size of each circle. Category scales of subjective area were obtained from each S's median category rating of each circle, while the magnitude estimation scales of area were the geometric means of responses to each circle. The differences between the category and magnitude estimation scales were typical of the difference commonly found between these two types of scales.

The scale values for number obtained for each S were related to the objective numbers (integers 1 to 10) by a negatively accelerated power function for both category and magnitude estimation instructions. A nonlinear relation for subjective numbers used in magnitude estimation is consistent with previous findings in supporting a two-stage model of magnitude estimation. However, the relation for numbers used in category

ratings was not consistent with previous results for category judgment. An analysis of the influence of the experimental procedure on the form of the number scales suggested that magnitude estimation instructions are a sufficient condition for a nonlinear relationship between subjective number and objective number. The analysis, however, also suggested that category instructions are not a sufficient condition for a nonlinear relationship between subjective number and objective number, i.e., the experimental task may influence the relationship between subjective and objective number. Hence, the similarity between the number scales obtained for category and magnitude estimation instructions was not considered as evidence that numbers are used in the same way for both types of judgments.

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Introduction

In the recent past, the identification of the source of the difference between category and magnitude estimation scales has been a dominant interest in psychophysics. The difference was convincingly demonstrated by Stevens and Galanter (1957). They showed that category and magnitude estimation scales were nonlinearly related. Category scales were found to be a negatively accelerated function of magnitude estimation scales. Stevens (1957) argued that when Ss assign stimuli to a finite number of categories their judgments are influenced by the discriminability of the stimuli and that discriminability is a decreasing function of the "true" sensory magnitudes. He concludes that category scales are invalid measures of sensory magnitudes.

An alternative position initiated by Garner (1954) and expanded by Attneave (1962) focused on the role of the numbers used as responses in quantitative judgment tasks. Garner argued that inherent in Steven's use of magnitude estimation procedures was the assumption that a S's numerical responses were valid measures of sensory magnitudes. He suggested that numbers used as responses may be nonlinear with the appropriate numerical representation of sensory magnitudes. Thus, Garner stimulated an interest in treating the use of numbers as the source of the difference between magnitude estimation and category scales.

Attneave (1962) carried Garner's notion a step further. He conceived the numerical representation of sensory magnitudes as a functional relationship between numerical responses and sensory magnitudes.

This elaboration of Garner's idea permitted Attneave to formulate an account of the difference between magnitude estimation and category scales in terms of different uses of numbers. Attneave proposed that a function relating stimulus magnitudes to judgments is not a simple function but rather a composite of an input and an output function. The input function represents the relationship between stimulus magnitudes and sensory magnitudes; and the output function represents the relationship between sensory magnitudes and numerical responses. The input function, if considered a power function, may be expressed mathematically as:

$$\Psi = a\phi^{\kappa}. \tag{1}$$

The output function, if considered a power function, may be expressed mathematically as:

$$J = b\psi^{m}. \tag{2}$$

Attneave claimed that viewing the psychophysical relationship as a composite permitted the prediction of the exponent for the composite function from the exponents of the two component functions. That is, since

$$\psi = a\phi^{k},$$

and $\psi = b'J^{1/m}$,

then $b'J^{1/m} = a\phi^{k}$

and thus $$J = (a/b')^{m}\phi^{km}. \tag{3}$$

Equation 3 may be simplified to

$$J = c\phi^{km} \tag{4}$$

in which $c = (a/b')^{m}$.

If $$J = c\phi^n \text{ (power law)}, \tag{5}$$

then $$\phi^{km} = \phi^n$$

or $$n = km. \tag{6}$$

Attneave's concept of the psychophysical relationship, referred to as the two-stage model, provided the conceptual framework for comparing the relationship between numerical responses and sensory magnitudes of category scales with that for magnitude estimation scales. Thus, Attneave provided an account for the difference between the two scales, in terms of the S's use of numbers.

The empirical separation of input and output exponents was accomplished by Curtis, Attneave, and Harrington (1968). Curtis, et al., found that the experimental procedure for judgments of differences lent itself to the mathematical operations necessary for the separation. In the 1968 study, Curtis, et al., assumed that both input and output functions could be described by a power function. They derived from Attneave's formulations the following expression for difference judgments.

$$J = a\,(\phi_1{}^k - \phi_2{}^k)^m + b, \tag{7}$$

where J is the difference judgments, a is a constant which represents the unit of measurement, and b is an additive constant which represents a displacement from the origin on the response scale. The additive constant accounts for response errors at the lower end of the function. The term ϕ, of course, represents the stimuli while k and m are the exponents for the input and output functions respectively. They obtained magnitude estimation judgments of the differences between a pair of weights and magnitude estimation judgments of weights

presented one at a time from a single group of *Ss*. The product of k and m predicted the exponent, n, for magnitude estimation: $k = .645$ while $m = 1.444$ and $k \times m = .735$, thus ñ is very close to the value of n, .746, obtained from magnitude estimation judgments of single weights. The study conducted by Curtis, *et al*., demonstrated the separation of the input and output exponents and provided empirical evidence in support of the contention that the parameters of input and output functions could be used to predict the exponent parameter of the composite psychophysical functions. However, since the estimation of difference was an uncommon task, the experiment raises the question of whether the results could have been due to the experimental procedure.

To investigate the generality of the findings, Curtis and Fox (1969) asked *Ss* to judge the combined sum of pairs of weights instead of differences between weights. They obtained sum judgments for both magnitude estimation and category ratings. The exponents for the input and output functions of the sum judgments were computed by the same equation as that used for difference judgments with the exception that the values were added instead of subtracted. The equation for judgments of sums was

$$J = a(\phi_1^{\,k} + {}_2^{\,k})^m + b. \qquad (8)$$

They found similar estimates of the input exponent k obtained from the two procedures and a difference in the output exponent m. The value of m for magnitude estimation was 1.20 as compared with 1.01 for category rating. This difference in output exponents was interpreted

as responsible for the curvilinear relation between category ratings and magnitude estimates. That is, there is a difference in the use of numbers for category and magnitude estimation judgments.

However, a comparison of the products, $\hat{n} = kxm$, for magnitude estimation and category judgments of sums shows little difference between these two estimates of the exponent, n. The values for magnitude estimation are $k = .657$, $m = 1.202$, and $kxm = .789$. The values for the category functions are $k = .720$, $m = 1.005$, and $kxm = .723$. The difference between .723 and .789 is too small to account for the curvilinear relation between category and magnitude judgments. Hence, the difference between category and magnitude estimation scales on the m parameter is inconclusive. The similarity of the predicted exponent from magnitude estimation of sums, obtained by Curtis and Fox, to that obtained by Curtis, et al. suggests that their magnitude estimation scale was not atypical. It is unfortunate that Curtis and Fox did not obtain category ratings of single stimuli in order to determine whether for their Ss the obtained exponent was similar to that predicted by the model and the usual curvilinear relation did not hold or the predicted and obtained exponents were dissimilar and the curvilinear relation held. With respect to why category scales are curvilinear with magnitude estimation scales, Curtis and Fox's results are equivocal.

A comparison of the exponent for number category and magnitude estimation judgments using the two-stage model was also made for difference judgments of brightness (Curtis, 1970). Four types of judgments were made by the Ss. The Ss first made magnitude estimation

judgments of difference in brightness of pairs of stimuli and magnitude estimation judgments of the brightness of single stimuli. The same *Ss* also made category judgments of differences in brightness of pairs of stimuli and category judgments of the brightness of single stimuli. These data provided better support for the hypothesis that the curvilinear relation between category scales and magnitude estimation scales reflects differences in how numbers are used in the two tasks. First, for single stimuli the usual curvilinear relation between scales for the two tasks held. The input exponents estimated from the difference judgments were similar (.16 for magnitude estimation and .11 for category ratings), while the output exponents differed (1.34 for magnitude estimation and .99 for category ratings). Furthermore, for each task the exponent from single-stimulus judgments was similar to the exponent predicted from the product of *k* and *m*. The predictions of the *n* exponent for individual *Ss* were very good for magnitude estimation judgments. The predictions of the *n* parameter, however, for the category judgments of individual *Ss* were not very good. There were large differences between the *ñ* obtained as a product of the *k* and *m* parameters and the *n* obtained from the category judgments of single stimuli. From these differences, in conjunction with a negative correlation between *k* and *m*, Curtis concluded that the assumptions of the two-stage model did not hold for category judgments of individual *Ss*. The data do not strongly support this conclusion, since the data was based on relatively few judgments. However, Curtis made the conclusion and chose to identify the assumption that a power function describes the output component as problematic.

There was some justification for the choice. Thus, the linearity exhibited by the output function for category judgments was considered an artifact. These results did not permit the identification of the form of the output function for category judgments nor did they clarify the influence of the experimental procedures on the construction of the category scales. The results do suggest, however, that the output function for category judgments may not be linear and that the output functions of individual Ss should be investigated before any general statements about category scales may be considered valid.

All of the studies designed to test the ability of the two-stage model to account for the discrepancy between category and magnitude estimation scales have not produced conclusive results. In particular they have not demonstrated that the difference between the two scales is a function of the way numbers are used in the two types of judgments. The studies, however, have shown that there is a great deal of variability in the use of numbers between Ss; and they have shown that the k and m parameters may yield predictions of n when there is a great deal of variability between Ss on m. The major empirical problems suggested by the studies are a determination of the invariance of the two-stage theory for different experimental procedures, establishment of a stable m parameter for category judgments, a comparison of the output functions for category and magnitude estimation judgments.

The purpose of the present study was to provide some empirical evidence about the form of scales of number and the influence of different types of judgments and experimental procedures on their

form. In order to accomplish this purpose, the study was designed to:
(1) Construct two kinds of scales for number; one for category instructions and one for magnitude estimation instructions.
(2) Obtain judgments by an experimental procedure different from that used in previous studies comparing these scales (Curtis, 1970; Curtis & Fox, 1969).
(3) Construct scales for individual Ss. Thus, the experiment may provide evidence of whether a different experimental procedure would produce number scales inconsistent with those obtained by other procedures; and whether Ss use numbers in the same way for both category and magnitude estimation judgments.

The experimental procedure was one employed by Rule (1969). The Ss compared the magnitudes of stimuli (circles) and numbers. From the frequency with which numbers were judged greater than circles, Thurstone-type discriminability scales of number were constructed for magnitude estimation and category rating instructions.

METHOD

Subjects

The subjects were six graduate students from the department of psychology at the University of Alberta. Two of the subjects were females and four were males. During the latter half of the experiment one of the male subjects was believed to be under excessive emotional tension.[1] After the last experimental session each subject was paid

[1] The subject's wife had been pregnant and undergone a difficult delivery with subsequent child-care problems.

$25.00 for participating in the experiment.

Apparatus

Two sets of 35-mm slides were prepared. Twelve slides consisted of different patterns of randomly positioned dots which varied in number. The number of dots in the various slides were: 13, 16, 20, 25, 31, 39, 49, 61, 76, 95, 119, and 149. Fifteen slides contained circles of different diameters. The diameters of the projected circles were .91, 1.05, 1.19, 1.36, 1.58, 1.84, 2.11, 2.44, 2.82, 3.26, 3.76, 7.34, 5.00, 5.78, and 6.68 in..

The stimuli appeared on the screen as white figures on a dark gray background. The images were projected onto a translucent screen via rear projection by a Selectroslide random-access projector with Somtar lens. Due to an equipment failure this projector was replaced by a Kodak Carousel (FA-950; remote control) projector with Ektamar zoom lens after three-fourths of the sessions had been completed. An exposure time of one second was controlled by an Alphax shutter.

The experiment was performed in a darkened room divided into two closed-off areas by a black plasterboard partition. Movement from one area to the other was facilitated by a door fixed in an opening to one side of the center point of the partition. A translucent screen, which was placed in an opening on the opposite side of the center, provided the surface for projection of the images. The _E_ and projector were on one side of the partition while the _S_ sat at a desk positioned immediately in front of the screen on the other side. The position of the desk and chair with respect to the

screen allowed the Ss to view the projected images at approximately eye level. The screen was 2 feet 5 1/2 inches above the floor and measured 3 feet 1 1/2 inches in width and 2 feet 3 1/2 inches in height. The projector was between 3 feet 2 inches and 3 feet 7 inches above the floor and projected an image at about the center of the screen. With the S sitting in a chair 1 foot 4 3/4 inches high and about 3 feet 6 inches from the screen the images were on nearly the same plane as the Ss' eyes.

Procedure

Each circle was paired with three or more of the integers one through ten. A total of 81 circle-number pairs were used. The pairs were selected such that the smaller circles were paired with the smaller integers and the larger circles with the larger integers. For example, circle number one (the smallest circle) was not paired with the integers four through ten while circle number fifteen (the largest circle) was not paired with those one through five. The 81 pairs were then arranged into 48 orders of presentation using a procedure of random selection with the restriction that neither the same circle or number occur in any two successive pairs. The specific selection of circle-number pairs was determined on the basis of data collected by Rule (1969).

Two males and one female were assigned to each of two groups. One group made circle-number estimations and estimations of circles (single stimulus presentation) under the equal-interval rating instructions, and the other group made circle-number anc circle estimations under magnitude estimation instructions. Twelve experimental sessions were conducted for each set of instructions.

The first session contained two parts. First, the _Ss_ were required to practice making judgments with one of the estimation procedures, (either category rating or magnitude estimation); in the second part, the _Ss_ began using the procedure to make circle-number estimations. The _E_ began the first session by explaining to the _S_ there would be two types of stimuli presented and that instructions for making estimations would be given before the presentation of each type of stimulus. The _E_ then read one of the following sets of instructions:

Magnitude Estimation

I am going to present a series of slides which contain patterns of dots. Your task is to assign numbers to the patterns of dots which correspond to your subjective impression of the number of dots in the pattern. The first slide I will present will be your standard and has been assigned the value of ten. You are to assign numbers to the remaining slides which are proportional to your subjective impression of the number of dots. For example, if the number of dots appears to be three times as large as the standard, assign a number three times as large as ten, that is 30. If the number of dots appears to be one-fifth as large as the standard, assign a number one-fifth as large as the standard that is two, and so on.

Each slide will be presented for only one second and you will have ten seconds between slides in which to record your judgments; so make each judgment as quickly as possible. Begin marking down your judgments on line one and continue down the column. Be sure you make a judgment for each slide. If you are not sure, simply guess.

The first slide is your standard. It has the value ten.

Equal-Interval Rating

I am going to present a series of slides which contain patterns of dots. Your task is to report your subjective impression of the number of dots in each pattern. In front of you is a rating scale. Notice that each line is numbered

> from one to ten. Under number one is written low and under number ten, high. If your subjective impression of the number of dots in a pattern is high, put an X on number ten; if it seems low, place an X on number one; if it is between high and low place an X on one of the remaining numbers. It is easiest if you consider each of these numbers as representing equal units of numerousness. So you can get oriented I will first present examples of number one and number ten. Your task is to judge the number of dots in the remaining slides by placing an X on one of the numbers on your rating scale. Use the first line for the second slide and so on.[2]

After the numerousness slides had been shown, a dim light was turned on and the S's response sheet collected. The E then explained that the second type of stimuli would be presented next and read one of the following sets of instructions:

Magnitude Estimation[3]

> Instead of your assigning numbers directly, your task will be to compare each circle with a number which I will present. After you have seen the standard, each time a circle is shown I will say a number. You are to tell me if the circle appears larger or smaller than the size which the number represents. For example, if I present a circle and say the number five, you would say larger if the circle appeared larger than one-half the size of the standard or you would say smaller if the circle appeared smaller than one-half the size of the standard. Remember the standard has been assigned the number ten. You must answer either larger or smaller to every circle and number presented. If you are not sure simply guess, but try to be as accurate as possible.

Equal-Interval Rating

> I am going to present a series of slides which contain circles. Your task is to report your subjective impression of the size of the circles. Again, the following is an explanation of the use of the rating scale in front of you.

---

[2]The remaining part of the Equal-Interval instructions was the same as the Magnitude Estimation instructions.

[3]The first paragraph of these instructions was the same as those for the numerousness only the stimuli were referred to as circles.

Notice, that it is numbered from one to ten. You are to consider these numbers as representing equal units of circle size. I will first present two slides which will be your standards. The first standard is a small circle and is an example of number one. The second standard will be a large circle and it is an example of number ten. I want you to consider the remaining circles as having been assigned numbers from one to ten which represent equal units of their size. For example, if you feel that a circle should be assigned the number four this would mean that the circle was three units larger than the small standard and six units smaller than the large standard. A circle to be assigned the number seven would need to be six units larger than the small standard and three units smaller than the large standard, and so on.

Instead of your assigning numbers directly, your task will be to compare each circle with numbers which I will present. After you have seen the two standards, each time a circle is shown I will say a number. You are to tell me if the circle appears larger or smaller than the size which the number represents. For example, if I present a circle and say the number five, you would say larger if the circle appeared more than five units larger than the lowest standard and you would say smaller if the circle appeared less than five units larger than the small standard. You must answer larger or smaller to every circle and number presented. If you are not sure simply guess, but try to be as accurate as possible.

Each _S_ was given the opportunity to ask questions about the instructions when the _E_ finished reading them. If the _S_ had no questions, the _E_ turned out the light and presented the standard(s) and two circle-number pairs which the _S_ estimated for practice.

The standard for the magnitude estimation condition was the largest circle and the standards for the rating condition were the smallest and largest. When the _S_ had given the appropriate responses of "smaller" and "larger" respectively to two practice pairs (Circle No. 3 was paired with the integer "9" and Circle No. 13 with the integer "2") the _E_ presented the standard(s) again and began the first of the

four orders of pairs of the session. Before each order was presented the standard(s) was shown and its numerical value called out.

The presentation of a circle-number pair consisted of the E loudly saying a number and showing the appropriate circle immediately afterward. After each circle image appeared on the screen the S responded by saying "larger" or "smaller". When the S had responded to the pairs of the first order, the E turned on a dim light and prepared for the presentation of the second order.

The eleven sessions which followed did not contain a practice session of numerousness judgments. The E used only the instructions and slides appropriate for the circle-number estimations.

A thirteenth experimental session was conducted to determine whether the Ss could make the appropriate judgments for unmodified category and magnitude estimation instructions. In this session, the Ss were required to make category or magnitude estimation judgments of circle size. Each of the fifteen circles was presented four times.

RESULTS

Three scales for category and three for magnitude estimation instructions were obtained from judgments of single stimuli (circles). The medians obtained from category judgments were fit to a power function by the method of least squares. Geometric means were computed from the magnitude estimation judgments. These means were also fit to a power function by the least squares method.

The function used to obtain the parameters of the power functions for both types of scales is represented by the equation:

$$Y = aX^{n}+b. \tag{9}$$

The terms _a_, _b_, and _n_ are constants. The term _a_ represents the value of the unit of each scale while _b_ represents a displacement from the origin. The term _n_ constitutes the exponent parameter of the power function as well as the most distinctive property of the function. These parameters are given in Table 1.

Table 1. Parameter values of functions obtained from judgments of single stimuli.

	Parameters $Y = aX^{n}+b$			
	Subject	a	b	n
	1	.49	-.30	1.64
Magnitude Estimation	2	.49	1.06	1.54
	3	.36	1.02	1.69
	4	.32	.89	1.76
Category Estimation	5	-296	297	- .02
	6	1.09	.61	1.08

The differences between the mean exponent of the category scales (.94) and that of the magnitude estimation scales (1.62) are representative of the nonlinear relationship found between these two types of scales.

The model employed to construct a scale of the subjective value

of number was an adaptation of the law of categorical judgment, Class I, Condition D (Torgerson, 1958). The law, as defined by Torgerson, is a set of equations which relate the parameters of stimuli and category boundaries to cumulative proportions derived from the proportion of times stimuli were placed in one or more of the categories which comprise a set of categories. The general form of the equation is:

$$t_g - s_j = X_{gj}(\sigma_g^2 + \sigma_j^2 - 2r_{gj}\sigma_g\sigma_j)^{\frac{1}{2}} \quad (10)$$

where t_g is the scale value for the g^{th} number, s_j is the scale value for the j^{th} circle, X_{gj} is the normal deviate corresponding to the proportion of times the j^{th} circle was judged smaller than the g^{th} number.

Torgerson's law of categorical judgment is fundamentally the same as Thurstone's law of comparative judgment. Torgerson's model is built upon Thurstone's concepts of discriminal processes, modal discriminal processes, discriminal differences, discriminal dispersions, and the assumption that discriminal dispersions describe normal distributions. The major difference lies in Torgerson's assumption that category boundaries may be treated as stimuli in the construction of a scale.

The general postulate represented by the equation above is that the proportion of times a judgment of "more than" or "less than" occurs is a consequence of the distance between two modal discriminal processes. The modal discriminal processes correspond to points on a psychological continuum of discriminal processes. Each modal process is represented by the scale values t_g and s_j and an interval scale was assumed representative of the continuum of discriminal processes. Thus, the difference $t_g - s_j$ expresses the amount of psychological distance between

a number (g) and a circle (j) on the continuum. The terms $\sigma_g{}^2$ and $\sigma_j{}^2$ within the radical represent the variances of the number g and circle j discriminal process distributions while $r_{gj}\sigma_g\sigma_j$ is their covariance. The radical, as a whole, is equal to the standard deviation of the distribution of differences.

The assumptions of condition D are that the discriminal dispersions are constant for number and for circle stimuli and the covariance terms were equal for each number-circle pair. Thus the above equation may be reduced to the form $t_g - s_j = X_{gj}C$, where C represents some constant value.

For the present analysis, the model differed from Torgerson's presentation in that t_g represented a scale value for a number rather than a category boundary. Second, proportions were obtained independently rather than cumulated across boundaries.

Three Thurstone-type discriminability scales of number were obtained from number-circle comparisons under magnitude estimation instructions and three such scales were obtained from number-circle comparisons under category instructions. The unit and origin of the scales were arbitrary. The scale values, however, are numerically representative of the distances between the values. Thus, the scales possess the properties of an interval scale.

The empirical scale values for category and magnitude estimation judgments are shown in Table 2. There was not sufficient information to estimate the scale value for integer 10 for S No. 1.

Table 2. Empirical scale values of discriminability scales of number.

	MAGNITUDE ESTIMATION			CATEGORY RATING		
	Subject					
Number	1	2	3	4	5	6
1	1	1	1	1	1	1
2	2.306	2.835	2.135	4.102	2.089	3.225
3	2.776	4.973	3.600	6.796	3.822	5.971
4	4.102	7.174	6.135	8.096	4.696	7.435
5	4.695	9.439	7.359	9.16	5.936	9.115
6	5.854	10.508	7.739	9.829	6.617	10.497
7	6.372	12.278	9.112	10.696	7.268	12.077
8	7.312	13.205	10.299	12.003	8.034	13.506
9	8.005	14.205	10.721	12.908	10.035	14.71
10		15.786	11.766	15.449	10.771	15.669

The empirical values of both sets of number scales were fit to a power function. The function is represented by the expression

$$Y = aX^{c} - b. \qquad (11)$$

The term c constitutes the exponent parameter of the power function and the value of the term 1/m of the output function. The method of least squares was used to fit the empirical scale values to the above power functions.

The fit of the predicted to the empirical scale values for number stimuli is displayed graphically in Figure 1. Plots for Ss 1-3 display the fits for magnitude estimation, and plots for Ss 4-6 display the fits for the category procedure.

In Figure 1, the empirical scale values are plotted against the predicted scale values and a straight line drawn through the points. Deviations from the line represent variations of the empirical from the predicted values. The plots in Figure 1 show that the predicted values fit the empirical rather well. Thus, the power function exponent parameter accurately represents the empirical function. The parameters for the above functions are given in Table 3.

The output component of a composite function which expresses a psychophysical relationship is often represented by the equation

$$J = a\psi^{m}. \tag{12}$$

However, the functional relationship determined for number in the present study is the inverse of equation (12). Since the subjective values of number were mapped onto number rather than the number stimuli mapped onto the subjective values, the equation which represents the functional relationship between the scale values of number and the stimuli is

$$\psi = a'J^{1/m}. \tag{13}$$

Where ψ represents the subjective value of a number and $\underline{J}$ the number stimuli (i.e., the integers 1 to 10). The term a' is equal to $(^{1}/_{a})^{1/m}$ which is equivalent to $\underline{a}$ in equation (12) and the exponent $\underline{1/m}$ is the reciprocal of $\underline{m}$ in equation (12). The constant $\underline{c}$ in the regression equation is equal to $\underline{1/m}$.

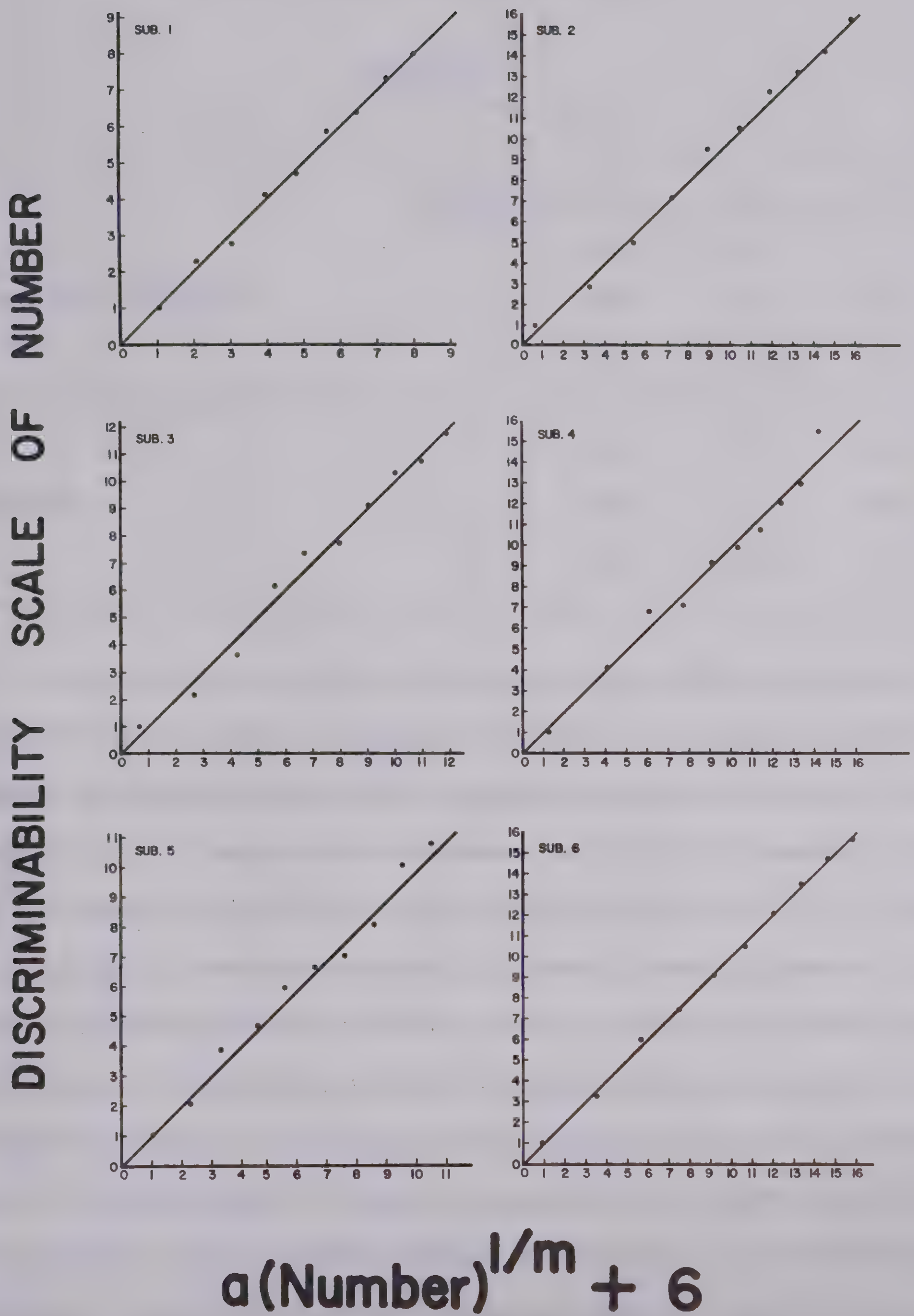

Figure 1. Scale values for number as a function of the predicted scale values for number (a (number)$^{1/m}$+b).

Table 3. Parameter values of functions obtained from number-circle comparisons.

Parameters
$Y = aX^{1/m} + b$

	Subject	a	b	1/m
	1	1.645	2.763	.775
MAGNITUDE ESTIMATION	2	4.994	5.334	.606
	3	4.094	4.455	.573
	4	9.332	9.048	.378
CATEGORY	5	1.569	1.512	.849
	6	5.388	5.486	.576

Table 3 shows there are relatively large differences between the exponents obtained for the category procedure. Although the differences between the magnitude estimation exponents are also substantial, they are not quite as large as those between the category exponents. There are two exponents which have essentially the same value, i.e., S3 and S6, although they were obtained by different judgment procedures. These large differences between exponents within a condition eliminate the possibility of identifying a single exponent common to the Ss of the category condition or to the Ss of the magnitude estimation condition. For all Ss, the value of the 1/m parameter for both types of procedures fall within a range of values defined by a negatively accelerated power function. Thus, the subjective values of number for both the magnitude

estimation and category procedures are related to objective numbers by a nonlinear function with a concave downward curvature.

A difference between the number scales for magnitude estimation and the scales for the category procedure appeared only in the variability among individual Ss. However, since the difference in variability did not alter the general form of the function for number, it may not be taken as indicative of a difference between the scales of the two conditions. Thus it appears that the number scales for the magnitude estimation and category procedures obtained in the present study are essentially the same.

DISCUSSION

The subjective number scales were related to objective number by a power function. The power function in each case had an exponent parameter of less than 1.0 (negatively accelerated). The similarity of the exponents indicate that the number scales obtained under magnitude estimation instructions were approximately the same as the number scales obtained under category estimation instructions.

Although there was some variance in the exponents for the individual Ss of the magnitude estimation condition, the exponents are in accordance with previous findings. The 1/m parameter obtained for magnitude estimation judgments in previous studies has varied considerably. In the studies which employed the experimental procedure for judgments of differences and sums (Curtis et al., 1968; Curtis & Fox, 1969; Curtis, 1970; Rule, Curtis, & Markley, 1970), the 1/m parameters

for magnitude estimation ranged from .6 to .9. In a study which constructed Thurstone-type scales (Rule, 1969), the 1/m parameter for the output function was different. A re-analysis of the data of Rule's (1969) study, reported by Rule, Curtis, Markley (1970), showed that the data obtained in the 1969 study fit a power function with an exponent of .4. A logarithmic function was originally reported. A study (Rule, et al., 1970) which employed a nonmetric scaling technique obtained a value of approximately .6 for the 1/m parameter for magnitude estimation judgments. The 1/m parameter estimates for the above studies define a range of values. The values range from .4 to .9. The estimates of the exponent for the number scale of magnitude estimation judgments in the present study fall well within the range defined by previous studies and support the findings that the output transformation for magnitude estimation is a negatively accelerated power function.

The variance in the exponents for individual Ss is also in accordance with previous findings. An analysis of the data for individual Ss in the studies conducted by Curtis et al. (1968) and Curtis (1970) showed considerable differences between exponents of individual Ss. In the former study the exponents, 1/m, ranged from .5 to 1.6.

Some of the previous studies which applied the two-stage model to magnitude estimation (Curtis, et al., 1968; Curtis & Fox, 1969) used different experimental procedures. Some of the studies designed to construct only a scale of number for magnitude estimation judgments have also used a different experimental procedure (Rule, 1969). All the number exponents obtained from pooled data in these studies fell within

the range of values prescribed by a negatively accelerated power function. A possible exception to these findings was reported by Rule (1972). Rule employed the method of triads to obtain a scale of number for magnitude estimation. Although the results obtained by Rule indicated that the subjective impression of number was related to objective number by a negatively accelerated function, there was some evidence that the function was inconsistent with a power function. However, the results of the present study, in conjunction with the findings of previous studies, show that the general form of the output function for magnitude estimation is relatively invariant for different experimental procedures and different scaling techniques.

Common to all studies were instructions to assign numbers proportional to the S's subjective magnitudes. This is the central feature of magnitude estimation instructions. Because the general form of the number scale (i.e., a negatively accelerated function of objective number) seems to be independent of the type of judgment task, the perceptual continuua, and the scaling technique, the magnitude estimation instructions appear to be a sufficient experimental condition for the nonlinear form of the relationship between subjective value of number and objective number. The implication of this relation is that the <u>Ss</u> may <u>not</u> be assumed to use numbers in magnitude estimations as though there were equal intervals between the numbers, e.g., they do not appear to use numbers in the same way as they would if they were permitted to use a ruler when judging the stimuli. The form of the relation indicates that the intervals between numbers vary in a systematic manner, i.e.,

as the numbers increase in value, the intervals between the scale values decrease in size. Such a variation in intervals implies that the proportional relationship ('twice as great') represented by such pairs of integers of '1' and '2', and '5' and '10' may not be represented by these integers when they occur as numerical responses in magnitude estimation judgments, but when such integers occur as numerical responses they may represent a 'less than' relationship between the subjective values they denote. That is, the value denoted by the numerical response '2' is less than twice as great as the value denoted by the numerical response '1'. Similarly, the value denoted by the numerical response '10' may not be twice as great as that value denoted by the numerical response '5' but instead it may be less than twice as great as the value denoted by the numerical response '5'. The extent to which such a relationship may be considered in more specific values such as 'half as much' or 'one and a half times greater' is not evident. It does seem evident, however, that the composite magnitude estimation scales may be biased by the S's use of numbers. Although the specific nature of the bias produced by the S's use of numbers may not be determined by the present results, the bias does appear to have a rather distinctive characteristic. Thus, although magnitude estimation scales may be distorted, they are distorted in about the same way.

In general, the results obtained for the magnitude estimation condition of the present study support the findings of investigations of Attneave's two-stage hypothesis, by Curtis, et al. (1968), Curtis & Fox (1969), Curtis (1970), and Rule, et al. (1970). The function

relating subjective value of number to objective number is consistent with the output function obtained in previous studies. Thus, the present results support the contention that an S's use of numbers may bias magnitude estimation scales. Since this contention is a fundamental part of the two-stage hypothesis, the present results support it as well.

The estimates of the exponent for subjective number, 1/m, for category judgments obtained in the present study are not consistent with those reported by Curtis and Fox (1969), and Curtis (1970). The results (obtained from pooled data) in both studies showed that sensory magnitudes and numerical responses for category judgments were related by a linear function. None of the individual functions for number which were obtained from category judgments in the present study, however, was linear. Instead, the exponents indicated a negatively accelerated function similar to those obtained from magnitude estimations. The inconsistency of these results complicate matters.

One of the primary questions relevant to the present study was whether the number function for category rating is the same as that for magnitude estimation or it is instead the case that the exponent of the number function for category ratings is larger (close to 1.0) than that of magnitude estimation. The present results indicate that the number scales are approximately the same, whereas the results of Curtis (1970) and Curtis and Fox (1969) indicate that the exponent of the number function for category rating is larger than that for magnitude estimation. If the present results are accepted they indicate that the nonlinear

relation between category and magnitude estimation scales is not due to differences in the number functions and that both scales are distorted in the same way by nonlinear number functions. Whereas Curtis' and Curtis and Fox's data indicate that although both scales may be biased measures of sensory magnitudes the differences between them is due to a difference in how Ss use numbers in each task. The present results alone do not provide much of a basis for deciding between the two alternatives.

The differences in the present data and those of Curtis (1970) and Curtis and Fox (1969) are in the category rating scales. The magnitude estimation results were similar. Although the experimental procedures used in the present study were different from those used by Curtis and Curtis and Fox, it is not uncommon in psychophysics to find that different techniques yield different scales. However, since the two procedures yielded similar results for magnitude estimation, the question pertinent to the above issues is why did the procedures yield different category scales and similar magnitude estimation scales?

The source of the similarity of the magnitude estimation scales and the difference between the category scales for the two procedures seems to lie in the demands imposed on the Ss by the judgment tasks. The demands of the magnitude estimation task were similar for both experimental procedures (that of Curtis, and Curtis and Fox; and that of the present study) while those of the category estimation tasks were different.

The magnitude estimation task of the previous studies provided

the Ss with a definite set of sensory impressions and relatively indefinite set of numerical responses (the set was defined only by the value of the standard and the restriction that all numbers be proportional to the standard's numerical value). The restrictions imposed by the magnitude estimation task in the present study were similar. The Ss were provided with a specific set of numbers and a relatively indefinite set of subjective values to associate with a number (the set of subjective values was defined only by the value given the standard and the requirement that all subsequent subjective values have a proportional relationship to the standard). The fact that in one case the Ss were selecting a subjective value to associate with a given number while in the other case they were selecting a number to associate with a given subjective impression apparently had no effect on the Ss' use of numbers.

The category task of the previous studies imposed restrictions on both the selection of numbers and the set of sensory impressions. The Ss were provided with a definite set of sensory impressions and a definite set of numbers. In addition to these restrictions, the Ss were required to assign numbers as though there were equal intervals between them. The category task in the present study did not impose as many restrictions. The Ss were provided with a relatively indefinite set of subjective values (the set was defined only by the values of the two standards and the restriction that the intervals between the values should be equal) and a definite set of numbers. Since the subjective values were essentially on a continuum, the Ss were permitted to make

as many divisions in the continuum as they believed necessary. Thus, the continuum was indefinite. The fact that the Ss were permitted to essentially define their continuum of subjective values in one case, and not in the other, may have had an effect on their use of numbers. Hence, the difference in the category number scales was produced by the different demands imposed by the tasks and the similarity of the magnitude estimation number scales was produced by the similarity of the demands imposed by the tasks. If the above account of the similarity and difference of the two number scales is in fact the case, it suggests that the elimination of restrictions on Ss' selection of subjective values for number is conducive to the use of numbers in accordance with the nonlinear function found in magnitude estimation.

In view of the above relationship between restrictions imposed by a task and the use of numbers, the similarity of the category and magnitude estimation number scales does not entail the conclusion that the use of numbers is the same in the two types of judgments. Since the task commonly involved in category judgments imposes restrictions which may attenuate the nonlinearity of a number scale and the category task in the present study did not impose such restrictions, the use of numbers may be different in the two types of judgments. The relationship between the restrictions imposed by a category task and the use of numbers suggest that the category instructions (i.e., emphasizing intervals between numbers rather than ratios) alone are not a sufficient condition for an approximately linear relationship between subjective value of number and objective number. Thus, the difference between

magnitude estimation and category scales may be a function of a different use of numbers.

The number scales obtained for magnitude estimation instructions were similar to those obtained in previous investigations of the output function for magnitude estimation. The results, in conjunction with previous findings, provide substantial evidence that the output function for magnitude estimation is a negatively accelerated power function. In addition, an analysis of the influence of the experimental procedures on the form of the number scales suggested the magnitude estimation instructions constitute a sufficient condition for the nonlinearity of the output function. These results lend support to the two-stage hypothesis.

The number scales obtained for category instructions were similar to those obtained for the magnitude estimation instructions. However, due to the probable influence of the category task on the form of the number scales, the similarity was not considered conclusive evidence that the number scales for category judgments are the same as those found for magnitude estimation. Thus, in regard to the question of whether the difference between category and magnitude estimation scales is due to a different use of numbers, the results are inconclusive. The results, however, do provide evidence that the category instructions are not a sufficient condition for a linear relationship between subjective value of number and objective number.

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